GUITAR PREAMP SIMULATION USING CONNECTION CURRENTS

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ABSTRACT

This paper deals with a method of decomposition of a nonlinear audio circuit based on so called connection currents. These currents are used to connect inner blocks of the audio circuit with regards to preserve mutual interaction between adjoined blocks. Although this approach requires usage of numerical algorithm to solve the nonlinear equations, it reduces number of nonlinear equations to be solved if the solution of inner blocks is approximated while the accuracy of simulation is comparable to numerical solution of the whole nonlinear audio circuit.

1. INTRODUCTION

Decomposition of more complex system to several, simpler blocks is usefull for implementation of digital audio effects in real-time. This decomposion is however essential for real-time simulation of complex nonlinear audio circuit, especially when more nonlinear circuit components are placed in the circuit. Many of different approaches of the decomposition were designed in preceding publications. Considering guitar amplifier simulation, a straightforward decomposition of the amplifier circuit was described in book [1] where the blocks were chosen according to the functional blocks in the circuit. Similar approach was used in [2] where real-time simulation of guitar distortion and overdrive effect pedals was discussed. In all cases, a combination of linear and nonlinear blocks builds the model of the whole audio circuit. But none of them considered interaction between adjoined blocks. The later paper [3, 4] showed that this interaction has impact to the accuracy of the simulation. In the same paper, the interaction between the blocks were included in the model using modified blockwise decomposition, where the second adjoined block served as a nonlinear load of the first block. The output signal from the first block was then fed again to the input of the second block with connected third block as the load. The biggest drawback of this method was the fact that the block serving as the nonlinear load was oversimplified to be able to approximate solution of both blocks connected together.

This problem was addressed in following paper [5] where the Fender guitar preamp core was the main objective. Although the preamp is build only by two triodes, the number of energy storing components is too high to allow the use approximation of nonlinear ordinary differential equations from the paper [3, 4] as well as for approximation of the equations derived from the DK-method [6]. It was found that for this type of circuit topology it is possible to decompose the nonlinear DK-method core into two parts due to some zero coefficients in the $\mathbf{K}$ matrix and solve them independently and subsequently do the state update of all state variables using one state update equation.

This method however fails when a local feedback is introduced - e. g. via Miller capacitances, which is the grid-to-plate capacitance multiplied by the tube amplification and which acts as an input capacitance and is resposible for higher frequencies roll-off. In that cases such a decomposition of the nonlinearity is not possible because all the $\mathbf{K}$ matrix coefficients are non-zero. However, majority of guitar preamps have feed-forward topology only with local feedbacks via the Miller capacitance. Individual blocks are therefore connected using one current flowing from the output from the first block and into the input of the second block. If we denote this connection current as an unknown variable and if we focus only at the solution of one block, then we are able to approximate solution of this block for input variables – input signal value, output signal value (connection current) and state variables. This approximation substitutes the numerical solving of the block, which can be computationally more efficient. The only task then is to find appropriate values of connection currents, which requires numerical solution but for less unknown variables than in the original circuit model.

2. GUITAR PREAMP MODEL

The guitar preamp Marshall JCM 800 has been chosen as a case study for the decomposition using the connection currents. The circuit schematic of the preamp is shown in Figure 1 [7]. Note that this is only a part of the preamp, the tone stack is omitted in this simulation because it is a linear circuit and is substituted by linear load connected to the node V_{out}. The circuit schematic consist of three tubes and therefore it will be divided into three blocks. The connection is denoted using the arrow with the label i_{conn1} resp. i_{conn2}, which are the unknown connection currents. The circuit further contains three capacitors C_{m1}, C_{m2} and C_{m3} modeling Miller capacitance which are responsible for the local feedback.

2.1. Derivation of Model Matrices

The proposed model of the preamp is based on state space description of the preamp circuit. During derivation of the model one can use automated DK-method using incidence matrices designed in [8] to get the model equations

$$\mathbf{x}[n] = \mathbf{A}\mathbf{x}[n-1] + \mathbf{B}\mathbf{u}[n] + \mathbf{C}\mathbf{i}_{\mathrm{n}}(\mathbf{v}[n]), \tag{1}$$
$$\mathbf{y}[n] = \mathbf{D}\mathbf{x}[n-1] + \mathbf{E}\mathbf{u}[n] + \mathbf{F}\mathbf{i}_{\mathrm{n}}(\mathbf{v}[n]), \tag{2}$$
$$\mathbf{v}[n] = \mathbf{G}\mathbf{x}[n-1] + \mathbf{H}\mathbf{u}[n] + \mathbf{K}\mathbf{i}_{\mathrm{n}}(\mathbf{v}[n]) \tag{3}$$

with DK-method matrices $\mathbf{A} \in \mathbb{R}^{10\times10}$, $\mathbf{B} \in \mathbb{R}^{10\times4}$, $\mathbf{C} \in \mathbb{R}^{10\times6}$, $\mathbf{D} \in \mathbb{R}^{1\times10}$, $\mathbf{E} \in \mathbb{R}^{1\times4}$, $\mathbf{F} \in \mathbb{R}^{1\times6}$, $\mathbf{G} \in \mathbb{R}^{6\times10}$, $\mathbf{H} \in \mathbb{R}^{6\times4}$, $\mathbf{K} \in \mathbb{R}^{6\times6}$, state vector $\mathbf{x}[n-1]$, input vector $\mathbf{u}[n]$ and nonlinear triode current models (grid and plate for each triode) in vector $\mathbf{i}_{\mathrm{n}}(\mathbf{v}[n])$. Obtaining the model matrices involves labeling circuit nodes (see Figure 1), deriving of incidence matrices (matrices which

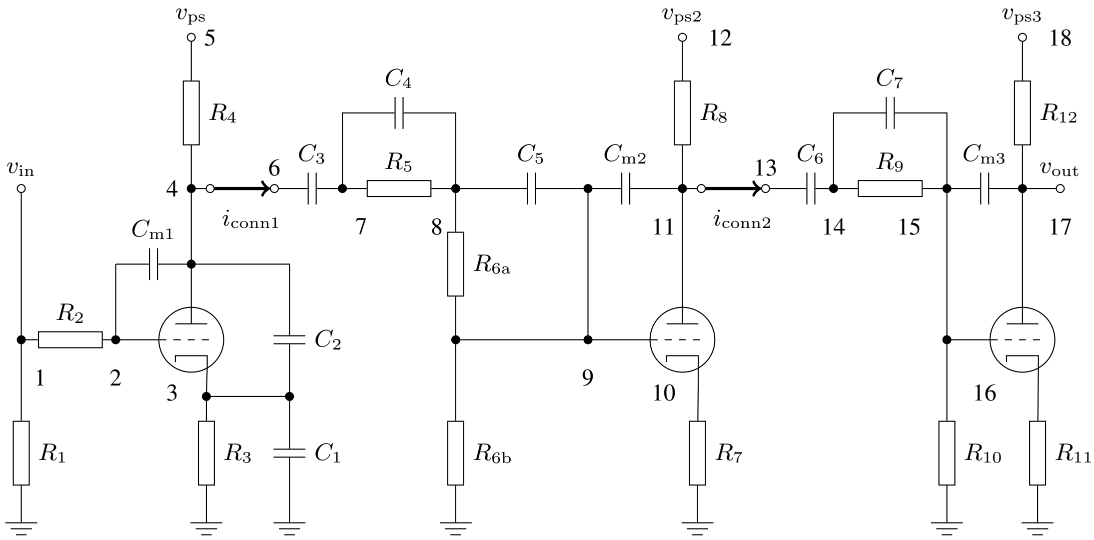

Figure 1: *Circuit schematic of the Marshall JCM 800 guitar preamp with the decomposition into blocks. The blocks are connected using connection currents i_{conn1} and i_{conn2}.*

Table 1: *Component values of the Marshall JCM 800 guitar preamp*

Component	Value	Component	Value	Component	Value
R_1, R_6	1 MΩ	R_2	68 kΩ	R_3	2.7 kΩ
R_4, R_8, R_{12}	100 kΩ	R_5, R_9, R_{10}	470 kΩ	R_7	10 kΩ
R_{11}	820 Ω	C_1	680 nF	C_2	100 pF
C_3, C_6	22 nF	C_4, C_7	470 pF	C_5	1 nF
C_{m1}, C_{m2}, C_{m3}	2 pF	V_{ps}	385 V	V_{ps2}	385 V

define connection of circuit elements to circuit nodes) and some algebra described in [8] to get the final matrices. Triode current models are defined as voltage controled current sources and one can choose different models e.g. Dempwols's model [9] or model based on interpolation above measured VA (Volt-Amper) characteristics. The output signal $\mathbf{y}[n]$ can be computed directly from (2) as well as the new state vector $\mathbf{x}[n]$ from (1). The only remaining problem is to solve the equation (3) which is in implicit form and requires numerical algorithm to solve six unknown variables.

This problem has been often solved by precomputation of the solution of equation (3) stored in multi-dimensional look-up tables and further interpolation of stored values [4, 5, 10, 11]. However, the dimension of six is too high considering either the required amount of data to be stored in a look-up table or time of precomputation. The decomposition of the equation (3) according to the paper [5] also fails here because all the $\mathbf{K}$ matrix coefficients are non-zero due to local feedbacks caused by capacitors C_{m1}, C_{m2} and C_{m3}.

Therefore, we further considered that each of blocks can be described using its own nonlinear equations and we used the unknown connection currents as additional inputs of the models. The new three circuit models have to be derived from the original circuit and each new model is described with equations in the same form as (1), (2) and (3) but with its own model matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, $\mathbf{E}$, $\mathbf{F}$, $\mathbf{G}$, $\mathbf{H}$ and $\mathbf{K}$ with subscript denoting block number. Each block also has its own state vector $\mathbf{x}$, inputs vector consisting of orignal inputs and connection currents, and nonlinear triode current model which now contains only one grid current model function and one plate current model function. The output signals and state update of the vectors $\mathbf{x}$ of each block can be computed directly from equations (2) and (1) and the remaining task is to solve the nonlinear equations, which are in this case given by

$$\mathbf{0} = \mathbf{G}_1\mathbf{x}_1 + \mathbf{H}_1 \begin{pmatrix} v_{\mathrm{in}} \\ v_{\mathrm{ps}} \\ i_{\mathrm{conn1}} \end{pmatrix} + \mathbf{K}_1\mathbf{i}_1(\mathbf{v}_1) - \mathbf{v}_1 \tag{4}$$

$$\mathbf{0} = \mathbf{G}_2\mathbf{x}_2 + \mathbf{H}_2 \begin{pmatrix} v_{\mathrm{ps2}} \\ i_{\mathrm{conn1}} \\ i_{\mathrm{conn2}} \end{pmatrix} + \mathbf{K}_2\mathbf{i}_2(\mathbf{v}_2) - \mathbf{v}_2 \tag{5}$$

$$\mathbf{0} = \mathbf{G}_3\mathbf{x}_3 + \mathbf{H}_3 \begin{pmatrix} v_{\mathrm{ps3}} \\ i_{\mathrm{conn2}} \end{pmatrix} + \mathbf{K}_3\mathbf{i}_3(\mathbf{v}_3) - \mathbf{v}_3 \tag{6}$$

for unknown variables $\mathbf{v}$ and matrices $\mathbf{G}$, $\mathbf{H}$ and $\mathbf{K}$ derived for each block independently. Then we can take the advantage of the DK-method: there are only two unknown controlling voltages - grid-to-cathode and plate-to-cathode voltage, no matter how many energy storing elements are contained in the circuit. The numerical algorithm can be replaced by two-dimensional look-up table and two-dimensional interpolation operating above the look-up table. It can be very efficient with regards to computational complexity as well as to size of the look-up table especially if the nonuniform grid is used [5].

In order to precompute the solution, the equations (4), (5), (6) have to be reformulated, e.g. the equation (4) can be rewritten into

$$\begin{aligned}
\mathbf{0} &= \mathbf{G}_1\mathbf{x}_1 + \mathbf{H}_1 \begin{pmatrix} v_{\text{in}} \\ v_{\text{ps}} \\ i_{\text{conn1}} \end{pmatrix} + \mathbf{K}_1\mathbf{i}_1(\mathbf{v}_1) - \mathbf{v}_1 = \\
&= \mathbf{G}_1\mathbf{x}_1 + \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \end{bmatrix} \begin{bmatrix} v_{\text{in}} \\ v_{\text{ps}} \\ i_{\text{conn1}} \end{bmatrix} + \\
&+ \mathbf{K}_1\mathbf{i}_1(\mathbf{v}_1) - \mathbf{v}_1 = \\
&= \mathbf{G}_1\mathbf{x}_1 + \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} v_{\text{in}} \\ v_{\text{ps}} \end{bmatrix} + \begin{bmatrix} h_{13} \\ h_{23} \end{bmatrix} i_{\text{conn1}} + \\
&+ \mathbf{K}_1\mathbf{i}_1(\mathbf{v}_1) - \mathbf{v}_1 = \\
&= \mathbf{p}_1 + \begin{bmatrix} h_{13} \\ h_{23} \end{bmatrix} i_{\text{conn1}} + \mathbf{K}_1\mathbf{i}_1(\mathbf{v}_1) - \mathbf{v}_1 = \\
&= \overline{\mathbf{p}_1} + \mathbf{K}_1\mathbf{i}_1(\mathbf{v}_1) - \mathbf{v}_1
\end{aligned} \tag{7}$$

where the vector $\overline{\mathbf{p}_1}$ contains two elements and therefore the equation (7) can be easily approximated as two-dimensional function $\mathbf{i}_{1\text{app}}(\overline{\mathbf{p}_1})$ where $\overline{\mathbf{p}_1} = \mathbf{p}_1 + \begin{bmatrix} h_{13} \\ h_{23} \end{bmatrix} i_{\text{conn1}}$. The equations (5) and (6) can be approximated in the similar way, both requiring the two dimensional approximation. The resulting solution related to grid-to-cathode current $i_{\text{g}} = f(v_{\text{gk}}, v_{\text{pk}})$ and plate-to-cathode current $i_{\text{p}} = f(v_{\text{gk}}, v_{\text{pk}})$ is stored in look-up tables. Size of grid current look-up table is 5 x 25 values and size of plate current look-up table is 14 x 49 values.

2.2. Connecting Blocks Together

Connecting blocks together via the connection currents means actually to find the unknown connection currents i_{conn1} and i_{conn2}. However, they cannot be found directly from (4), (5), (6) because there are only two unknown variables in three equations and therefore further conditions are required to find the currents. We can exploit the fact that voltages in adjoined nodes v_4, v_6 resp. v_{11}, v_{13} (see Figure 1) have to be equal. These voltages can be extracted from output equation (5) but with matrices for given block. The voltage v_4 can be expressed as

$$v_4 = \mathbf{D}_{11}\mathbf{x}_1 + \mathbf{E}_{11} \begin{pmatrix} v_{\text{in}} \\ v_{\text{ps}} \\ i_{\text{conn1}} \end{pmatrix} + \mathbf{F}_{11}\mathbf{i}_1(\mathbf{v}_1). \tag{8}$$

The second block has two terminals with voltages

$$v_6 = \mathbf{D}_{21}\mathbf{x}_2 + \mathbf{E}_{21} \begin{pmatrix} v_{\text{ps2}} \\ i_{\text{conn1}} \\ i_{\text{conn2}} \end{pmatrix} + \mathbf{F}_{21}\mathbf{i}_2(\mathbf{v}_2), \tag{9}$$

$$v_{11} = \mathbf{D}_{22}\mathbf{x}_2 + \mathbf{E}_{22} \begin{pmatrix} v_{\text{ps2}} \\ i_{\text{conn1}} \\ i_{\text{conn2}} \end{pmatrix} + \mathbf{F}_{22}\mathbf{i}_2(\mathbf{v}_2) \tag{10}$$

and the connection node of the third block is

$$v_{13} = \mathbf{D}_{31}\mathbf{x}_3 + \mathbf{E}_{31} \begin{pmatrix} v_{\text{ps3}} \\ i_{\text{conn2}} \end{pmatrix} + \mathbf{F}_{31}\mathbf{i}_3(\mathbf{v}_3). \tag{11}$$

Conditions $v_4 = v_6$ and $v_{11} = v_{13}$ lead to the final equations

$$\begin{aligned}
0 &= \mathbf{D}_{11}\mathbf{x}_1 + \mathbf{E}_{11} \begin{pmatrix} v_{\text{in}} \\ v_{\text{ps}} \\ i_{\text{conn1}} \end{pmatrix} + \mathbf{F}_{11}\mathbf{i}_1(\mathbf{v}_1) \\
&- \mathbf{D}_{21}\mathbf{x}_2 - \mathbf{E}_{21} \begin{pmatrix} v_{\text{ps2}} \\ i_{\text{conn1}} \\ i_{\text{conn2}} \end{pmatrix} - \mathbf{F}_{21}\mathbf{i}_2(\mathbf{v}_2)
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
0 &= \mathbf{D}_{22}\mathbf{x}_2 + \mathbf{E}_{22} \begin{pmatrix} v_{\text{ps2}} \\ i_{\text{conn1}} \\ i_{\text{conn2}} \end{pmatrix} + \mathbf{F}_{22}\mathbf{i}_2(\mathbf{v}_2) - \\
&- \mathbf{D}_{31}\mathbf{x}_3 - \mathbf{E}_{31} \begin{pmatrix} v_{\text{ps3}} \\ i_{\text{conn2}} \end{pmatrix} - \mathbf{F}_{31}\mathbf{i}_3(\mathbf{v}_3)
\end{aligned} \tag{13}$$

from which the connection currents i_{conn1} and i_{conn2} can be numerically computed. Considering precomputation of all blocks (4), (5), (6) it is possible to compute nonlinear currents flowing through triodes directly using approximating functions $\mathbf{i}_{1\text{app}}(\mathbf{p_1})$, $\mathbf{i}_{2\text{app}}(\mathbf{p_2})$ and $\mathbf{i}_{1\text{app}}(\mathbf{p_3})$. These approximating functions can be substituted into (12) and (13) to get equations

$$\begin{aligned}
0 &= \mathbf{D}_{11}\mathbf{x}_1 + \mathbf{E}_{11} \begin{pmatrix} v_{\text{in}} \\ v_{\text{ps}} \\ i_{\text{conn1}} \end{pmatrix} + \\
&+ \mathbf{F}_{11}\mathbf{i}_{1\text{app}} \left(\mathbf{p}_1 + \begin{bmatrix} h_{13} \\ h_{23} \end{bmatrix} i_{\text{conn1}} \right) - \\
&- \mathbf{D}_{21}\mathbf{x}_2 - \mathbf{E}_{21} \begin{pmatrix} v_{\text{ps2}} \\ i_{\text{conn1}} \\ i_{\text{conn2}} \end{pmatrix} - \\
&- \mathbf{F}_{21}\mathbf{i}_{2\text{app}} \left(\mathbf{p}_2 + \begin{bmatrix} h_{12} & h_{13} \\ h_{22} & h_{23} \end{bmatrix} \begin{bmatrix} i_{\text{conn1}} \\ i_{\text{conn2}} \end{bmatrix} \right),
\end{aligned} \tag{14}$$

$$\begin{aligned}
0 &= \mathbf{D}_{22}\mathbf{x}_2 + \mathbf{E}_{22} \begin{pmatrix} v_{\text{ps2}} \\ i_{\text{conn1}} \\ i_{\text{conn2}} \end{pmatrix} + \\
&+ \mathbf{F}_{22}\mathbf{i}_{2\text{app}} \left(\mathbf{p}_2 + \begin{bmatrix} h_{12} & h_{13} \\ h_{22} & h_{23} \end{bmatrix} \begin{bmatrix} i_{\text{conn1}} \\ i_{\text{conn2}} \end{bmatrix} \right) - \\
&- \mathbf{D}_{31}\mathbf{x}_3 - \mathbf{E}_{31} \begin{pmatrix} v_{\text{ps3}} \\ i_{\text{conn2}} \end{pmatrix} - \\
&- \mathbf{F}_{31}\mathbf{i}_{3\text{app}} \left(\mathbf{p}_3 + \begin{bmatrix} h_{13} \\ h_{23} \end{bmatrix} i_{\text{conn2}} \right).
\end{aligned} \tag{15}$$

The simulation model now only requires numerical solving in real-time for two unknowns – $i_{\text{conn1}}, i_{\text{conn2}}$.

The final computational scheme is:

1. compute input vector $\mathbf{p}[n] = \mathbf{G}\mathbf{x}[n-1] + \mathbf{H}\mathbf{u}[n]$,
2. compute connection currents using Newton-Raphson algorithm from equations (14) and (15),
3. compute nonlinear triode currents using approximated equations (4), (5) and (6) ,
4. compute output signal from (2) and
5. update state vector using (1).

3. RESULTS

Two properties of the new simulation model were investigated – computational complexity savings and error between original model and model with block decomposition and connection currents. Both models – original DK-model without the decomposition and new proposed model – were implemented in C++ language as a mex library for Matlab. Signals with recorded guitar riff were used as testing signals. The standard audio sampling rate 48 kHz was used in this case, but an oversamling is required for higher input levels to improve numerical stability and to prevent the aliasing distortion. Because in ideal case, the output signals from both models should be the same, comparison of signals in time-domain was used. A part of the output signal for both models and their error signals (difference) are shown in Figure 2. The results show very good match between the output signals – both output signals are visibly overlapped, their deviation is lower than 0.1 % related to the maximum of the output signal. The deviation is caused foremost by different numerical scheme and partly by approximation of solution of all three blocks.

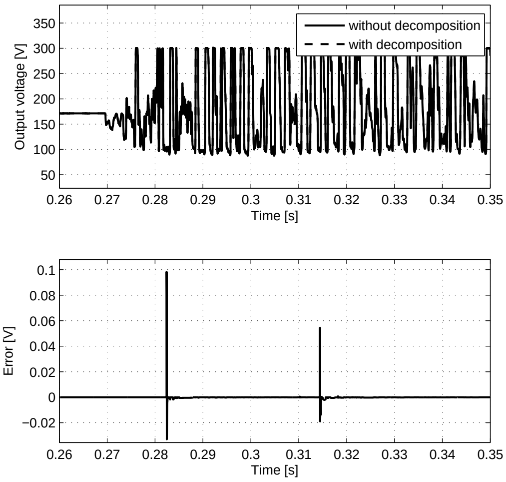

Figure 2: *Comparison of output signals in time domain. The dashed line is overlapped with the solid and therefore their difference is also shown (bottom).*

The second investigated property was computational complexity savings. The original model without the decomposition was not able to run in real-time and computational complexity was strongly dependent on input signal level. The new model with decomposition into blocks consumed between 10 and 15 % of CPU on Intel 2.66 GHz processor and is also dependant on input signal level because this algorithm also requires numerical algorithm. This algorithm is however capable of running in real-time with comparable accuracy of the simulation.

4. CONCLUSIONS

The new block decomposition of more complex nonlinear audio circuits has been discussed in this paper. This block decomposition preserves mutual interaction between the adjacent blocks, which leads to accuracy of the simulation comparable to numerical solving of the whole circuit without any decomposition into blocks. The drawback of this method is that it still requires usage of numerical algorithm to solve the equations but if the inner blocks are approximated, then the number of unknown variables to be solved numerically is equal to number of connection currents, which is much lower than original unknown variables. The algorithm was tested on simulation of Marshall JCM 800 guitar preamp and the results showed that the algorithm is capable of running in real-time.

5. ACKNOWLEDGMENTS

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6. REFERENCES

[1] U. Zölzer, *DAFX - Digital Audio Effects*, chapter Nonlinear Processing, p. 602, J. Wiley & Sons, Ltd, New York, 2011.

[2] D. T. Yeh, J. S. Abel, and J. O. Smith, “Simplified, physically-informed models of distortion and overdrive guitar effects pedal,” in *Proc. Digital Audio Effects (DAFx-07)*, Bordeaux, France, Sep. 10-15, 2007, pp. 189–196.

[3] J. Mačák and J. Schimmel, “Real-time guitar tube amplifier simulation using approximation of differential equations,” in *Proceedings of the 13th International Conference on Digital Audio Effects DAFx10*, Graz, Austria, Sep. 6-10, 2010.

[4] J. Mačák and J. Schimmel, “Real-time guitar preamp simulation using modified blockwise method and approximations,” *EURASIP Journal on Advances in Signal Processing*, vol. 2011, pp. 11, 2011.

[5] J. Mačák, M. Holters, and J. Schimmel, “Simulation of a vacuum-tube push-pull guitar power amplifier,” in *Proceedings of the 15th International Conference on Digital Audio Effects DAFx12*, York, United Kingdom, Sept. 17–21,, 2012.

[6] D. T. Yeh, *Digital Implementation of Musical Distortion Circuits by Analysis and Simulation*, Ph.D. thesis, Center for Computer Research in Music and Acoustics, Stanford, 2009.

[7] “Schematic Heaven,” [online], Available from http://www.webphix.com/schematic20heaven/ www.schematicheaven.com/index_HTML.html, 2000.

[8] M. Holters and U. Zölzer, “Physical modelling of a wah-wah effect pedal as a case study for application of the nodal dk method to circuits with variable parts,” in *Proceedings of the 14th International Conference on Digital Audio Effects DAFx11*, Paris, France, Sept. 19–23,, 2011.

[9] K. Dempwolf and U. Zölzer, “A physically-motivated triode model for circuit simulations,” in *Proc. Digital Audio Effects (DAFx-11)*, Paris, France, Sept. 19–23, 2011.

[10] D. T. Yeh, J. S. Abel, and J. O. Smith, “Automated physical modeling of nonlinear audio circuits for real-time audio effects - Part I: Theoretical development,” *IEEE Trans. Audio, Speech, and Language Processing*, vol. 18, no. 4, pp. 728–737, May 2010.

[11] J. Pakarinen and M. Karjalainen, “Wave digital simulation of a vacuum-tube amplifier,” in *Proc. Intl. Conf. on Acoustics, Speech, and Signal Proc.*, Toulouse, France, May 15–19, 2006, pp. 153–156.